

Evolution of the Potential in Cosmological Gravitational Clustering

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Abstract

In recent years there has been a developing realization that the interesting large-scale structure of voids, “pancakes”, and filaments in the Universe is a consequence of the efficacy of an approximation scheme for cosmological gravitation clustering proposal by Zel’dovich in 1970. However, this scheme was only supposed to apply to smoothed initial conditions.

We show that this can be explained by the fact that the gravitationally evolved potential from N-body simulations closely resembles the smoothed potential of the initial conditions. The resulting “hierarchical pancaking” picture effectively combines features of the former Soviet and Western theoretical pictures for galaxy and large-scale structure formation.

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Over the last forty years, it has been realized that galaxies are clustered in space, which is measurable by a two-point correlation function [1]. During the last twenty years, it has become clear that the distribution has a rich, frothy structure of voids, filaments, and sheets [2]. Gravitational instability is the dominant theory for explaining the development of clumpy structure in the Hot Big Bang Cosmology. Yet there is a problem in accounting for all the coherent anisotropic structure.

In 1970 Zel'dovich [3] proposed an approximation for clustering which is essentially inertial motion in comoving coordinates. Specialized to critical density models, it is

$$\bar{x}(\bar{q}, t) = \bar{q} + a(t) \nabla S(\bar{q}) \quad (1)$$

where \bar{x} is the (Eulerian) position, \bar{q} is the initial (Lagrangian) coordinate, a is the scale factor of the Universe, and S is a velocity potential related by a constant factor to the gravitational potential ϕ . Divergence in ϕ is prevented by of doing the convention change of variables to density contrast $\frac{\rho - \bar{\rho}}{\bar{\rho}}$ as the source term for the Poisson equation. The pseudo-Newtonian treatment of cosmological gravitational clustering has recently been put on a firm footing [4,5]

In spite of the general success of the gravitational instability picture, there is no ready explanation for the coherence of large scale structure, i.e superclusters. The Zel'dovich approximation (ZA) does predict such structures [6], but ZA depends on the assumption of long-range coherence of the gravitational potential (otherwise particles should be deflected from their trajectories by very small-scale inhomogeneities). Theories with smoothed initial conditions, such as ‘‘Hot Dark Matter’’, apparently have trouble making any structures in voids, which results in an excessive correlation amplitude [7].

Hierarchical clustering models, which have initial density fluctuations on all scales, are more generally successful. Such models are typically specified by the power spectrum of density fluctuations

$$P(k) \equiv \langle \delta_k^2 \rangle \quad (2)$$

where the δ_k are Fourier components. Power laws

$$P(k) \propto k^n \tag{3}$$

are useful for theoretical analysis and discussion. The most favored models today have $n \sim 1$ for small k , possibly a relic of inflation, and turn over gradually to negative n at large k , depending on the matter contents of the Universe. Such models usually are based on collisionless dark matter.

In spite of the fact that these theories do not have smoothed initial conditions, evidence began accumulating from numerical simulations that they produced interesting large-scale anisotropic structures [8,9]. As this became an accepted feature of such models, an explanation was developed based on the adhesion approximation [10,11]. This explanation contains the correlation length of ϕ as a crucial feature, and consequently identifies $n = -1$ as a transitional power law.

However, this argument must be incomplete. Use of ZA can predict structure very well if the initial conditions are smoothed, removing small-scale initial power. The best smoothing appears to be Gaussian convolution around the scale of nonlinearity [12]. More recently, a detailed study of the behavior of the approximation (which we call TZA for truncation of the spectrum, to distinguish it from ZA) over a range $-3 \leq n \leq +3$ has been made with surprising results [13]. The performance of TZA degrades as n increases but is still quite good even for the extreme case $n = +3$; it is far better than conventional Eulerian linear perturbation theory for example. One can use the initial conditions to predict the location and orientation of filament-like objects with considerable accuracy. N-body simulations appear to be no longer necessary for most spectra of interest if one is satisfied with resolving galaxy group mass scales [12].

An explanation is now possible for the unreasonable utility of TZA; more detail is presented elsewhere [13]. Because the growth of fluctuation amplitude compensates for expansion, the peculiar gravitational potential evolves to linear order such that ϕ is constant. We discuss for changes beyond linear order.

In Figure 1 we show the initial and nonlinear evolved potential along a diagonal of each of four simulation cubes. We did power-law N-body simulations with $n = -3, -1, +1$, and $+3$. These are 128^3 PM simulations [14]. The moment chosen is that when the scale of nonlinearity has grown to $k_{nl} = 8k_f$, where k_f is the fundamental mode of the cube.

For $n = -3$ and -1 the evolved potential is very close to the initial, as expected from trivial considerations (the power spectrum of the potential is $n - 4$, so long waves dominate). For $n = +1$ and $+3$ there is no real resemblance between the potentials, although the eye can detect some correlation for $n = +1$.

Computation of the coherence length of the potential shows that it is largely unchanged by evolution for $n = -3$ and -1 . However it grows by a factor of more than 7 for $n = +1$ and $+3$. (In both cases the number should be larger; we are resolution limited, especially for $+3$). Adhesion arguments are based on coherence of the initial potential, and so missed making the prediction in the latter two cases.

Now let us compare the evolved potential to the smoothed initial potential. This is shown in Figure 2 with a scale change for clarity. There is a strong resemblance, decreasing with n . It is clear from this that modestly nonlinear evolution (up to $\delta_\rho/\rho \sim 1$ from linear extrapolation) can be accurately described by constant gravitational potential with increasing smoothing. We have crosscorrelated the initial with the final potential. There is a significant signal, increasing predictably as n decreases. Modes $k < k_{nl}$ are known to grow linearly. However, when we smooth the initial potential by Gaussian convolution, there is an enormous increase in signal, verifying the visual impression of comparing Figures 1 and 2. The smoothing windows found to work best for TZA [12] were used for this comparison. In the Table we show further information. More details on this and other detailed comparisons between the simulations at TZA are given elsewhere [13].

The very strong resemblance of the nonlinear potential to the smoothed initial potential explains why TZA works so well over such a wide range of spectral indices. The clumps which have formed by hierarchical clustering are moving in a background potential which is close to a smoothed version of the initial. Also, adhesion and other approximations are

limited [15] by their use of the initial potential. Even for $n = -1$ we find some improvement by smoothing. This suggests that a new class of second-order approximation schemes can be constructed which go beyond ZA but use the smoothed potential.

The Universe possesses a rich large-scale structure because gravitational clustering smooths the potential. This is a somewhat counter-intuitive result, but there have been precursor hints, for example based on the topology of large-scale structure [16]. Furthermore, the smoothed and linearly evolved density field manifestly does not resemble the nonlinear density field [17]. Hierarchical clustering (largely developed in the West) is a good description of small-scale clumping and gives reasonable results for galaxy formation [2]. However, the motion of the clumps is driven by the smoothed potential, which brings into play all the machinery developed by the “Moscow school” during the 70’s and 80’s [6]. It appears that a unified picture of galaxy and large-scale structure formation can now emerge.

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FIGURE CAPTIONS

Fig. 1. The peculiar gravitational potential is shown along one diagonal of each of the four simulation cubes for (a) $n = -3$ (b) $n = -1$ (c) $n = +1$ (d) $n = +3$. The units are arbitrary but do reflect the fact that the potential is constant to linear order. The dotted line is the initial potential is constant to linear order. The dotted line is the initial potential and the solid line the evolved. Note strong evolution for $n \geq 1$.

Fig. 2. The potential is plotted in the same way as Figure 1 except that the initial potential is smoothed by Gaussian convolution, and the vertical scale has been expanded for $n = +1, +3$.

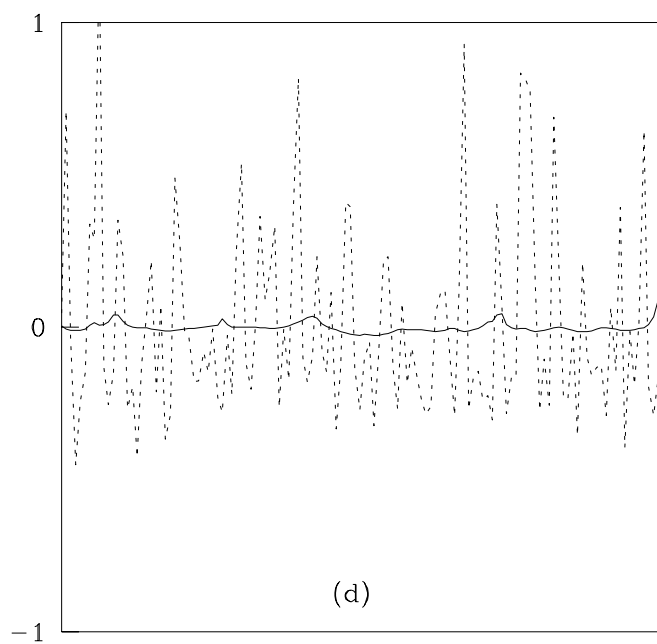
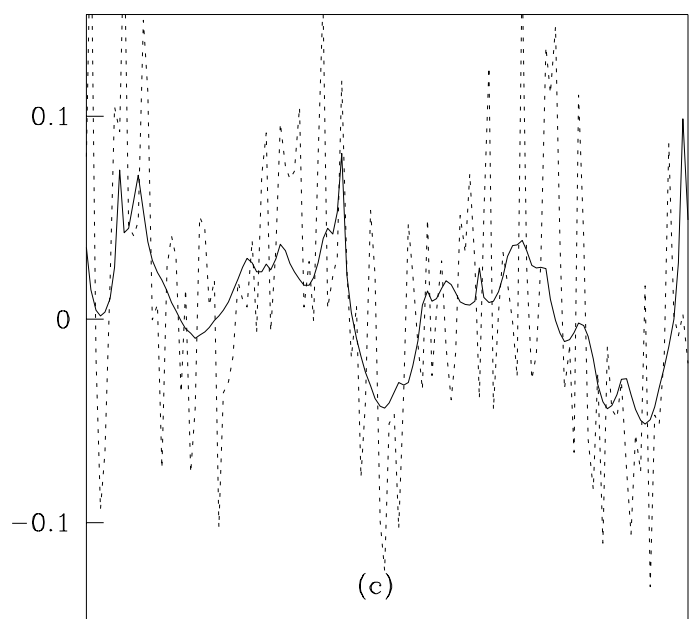
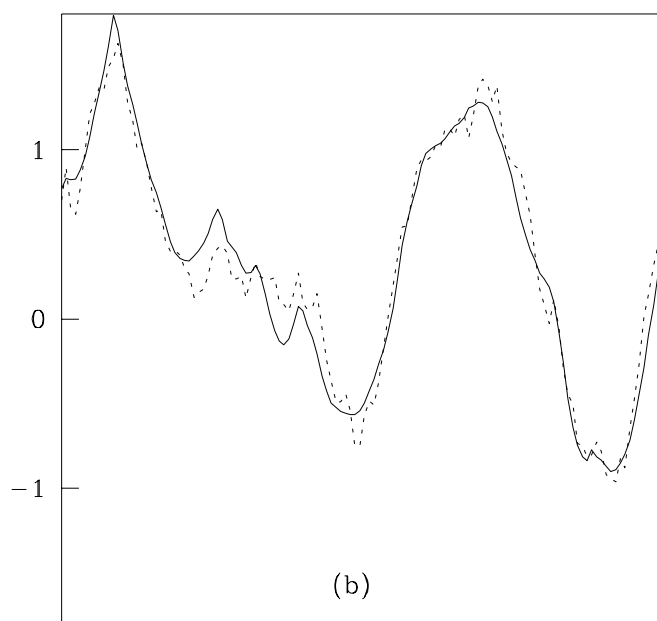
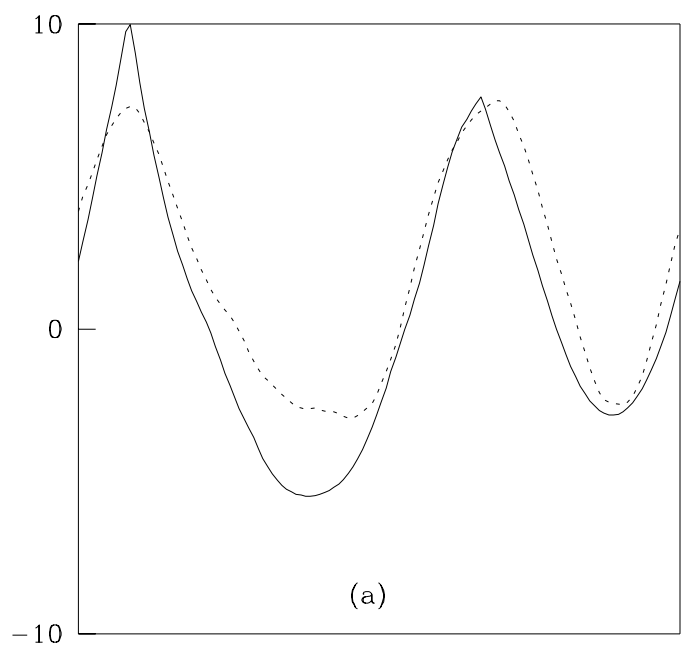
TABLES

TABLE I. Cross-correlation of gravitational potentials.

Spectral Index	Smoothed Initial/Initial	Initial/Final	Smoothed Initial/Final
-3	–	0.96	–
-1	0.999	0.987	0.990
+1	0.68	0.65	0.94
+3	0.14	0.10	0.69

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